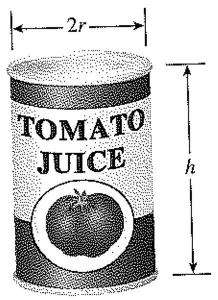


FIGURE 2.62 A tomato juice can. (Example 7)

Solve Graphically

The graph of P in Figure 2.61 shows a minimum of approximately 56.57, occurring when $x \approx 14.14$.

Interpret

The width is about 14.14 m, and the minimum perimeter is about 56.57 m. Because $200/14.14 \approx 14.14$, the dimensions of the rectangle with minimum perimeter are 14.14 m by 14.14 m, a square.

Now try Exercise 31.

EXAMPLE 7 Designing a Juice Can

Stewart Cannery will package tomato juice in 2-liter cylindrical cans. Find the radius and height of the cans if the cans have a surface area of 1000 cm^2. (See Figure 2.62.)

SOLUTION

Model

S = surface area of can in cm^2

r = radius of can in centimeters

h = height of can in centimeters

Using volume (V) and surface area (S) formulas and the fact that 1 L = 1000 cm^3, we conclude that

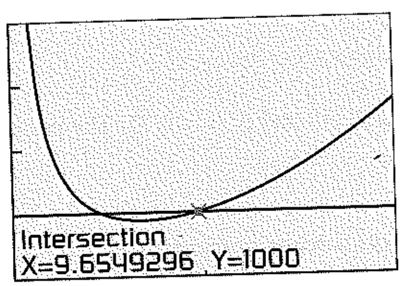
$$V = \pi r^2 h = 2000 \quad \text{and} \quad S = 2\pi r^2 + 2\pi r h = 1000.$$

So

$$2\pi r^2 + 2\pi r h = 1000$$

$$2\pi r^2 + 2\pi r\left(\frac{2000}{\pi r^2}\right) = 1000 \qquad \text{Substitute } h = 2000/(\pi r^2).$$

$$2\pi r^2 + \frac{4000}{r} = 1000 \qquad \text{Equation to be solved}$$



[0, 20] by [0, 4000]

FIGURE 2.63 (Example 7)

Solve Graphically

Figure 2.63 shows the graphs of $f(x) = 2\pi r^2 + 4000/r$ and $g(x) = 1000$. One point of intersection occurs when r is approximately 9.65. A second point of intersection occurs when r is approximately 4.62.

Because $h = 2000/(\pi r^2)$, the corresponding values for h are

$$h = \frac{2000}{\pi(4.619...)^2} \approx 29.83 \text{ and } h = \frac{2000}{\pi(9.654...)^2} \approx 6.83.$$

Interpret

With a surface area of 1000 cm^2, the cans either have a radius of 4.62 cm and a height of 29.83 cm or have a radius of 9.65 cm and a height of 6.83 cm.

Now try Exercise 33.

QUICK REVIEW 2.7 *(For help, go to Sections A.3. and P.5.)*

In Exercises 1 and 2, find the missing numerator or denominator.

1. $\frac{2x}{x-3} = \frac{?}{x^2+x-12}$ **2.** $\frac{x-1}{x+1} = \frac{x^2-1}{?}$

In Exercises 3–6, find the LCD and rewrite the expression as a single fraction reduced to lowest terms.

3. $\frac{5}{12} + \frac{7}{18} - \frac{5}{6}$ **4.** $\frac{3}{x-1} - \frac{1}{x}$

5. $\frac{x}{2x+1} - \frac{2}{x-3}$

6. $\frac{x+1}{x^2-5x+6} - \frac{3x+11}{x^2-x-6}$

In Exercises 7–10, use the quadratic formula to find the zeros of the quadratic polynomials.

7. $2x^2 - 3x - 1$ **8.** $2x^2 - 5x - 1$

9. $3x^2 + 2x - 2$ **10.** $x^2 - 3x - 9$

SECTION 2.7 EXERCISES

In Exercises 1–6, solve the equation algebraically. Support your answer numerically and identify any extraneous solutions.

1. $\frac{x-2}{3} + \frac{x+5}{3} = \frac{1}{3}$ **2.** $x + 2 = \frac{15}{x}$

3. $x + 5 = \frac{14}{x}$ **4.** $\frac{1}{x} - \frac{2}{x-3} = 4$

5. $x + \frac{4x}{x-3} = \frac{12}{x-3}$ **6.** $\frac{3}{x-1} + \frac{2}{x} = 8$

In Exercises 7–12, solve the equation algebraically and graphically. Check for extraneous solutions.

7. $x + \frac{10}{x} = 7$ **8.** $x + 2 = \frac{15}{x}$

9. $x + \frac{12}{x} = 7$ **10.** $x + \frac{6}{x} = -7$

11. $2 - \frac{1}{x+1} = \frac{1}{x^2+x}$ **12.** $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$

In Exercises 13–18, solve the equation algebraically. Check for extraneous solutions. Support your answer graphically.

13. $\frac{3x}{x+5} + \frac{1}{x-2} = \frac{7}{x^2+3x-10}$

14. $\frac{4x}{x+4} + \frac{3}{x-1} = \frac{15}{x^2+3x-4}$

15. $\frac{x-3}{x} - \frac{3}{x+1} + \frac{3}{x^2+x} = 0$

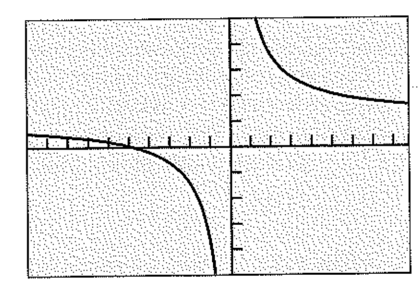
16. $\frac{x+2}{x} - \frac{4}{x-1} + \frac{2}{x^2-x} = 0$

17. $\frac{3}{x+2} + \frac{6}{x^2+2x} = \frac{3-x}{x}$

18. $\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x^2+3x}$

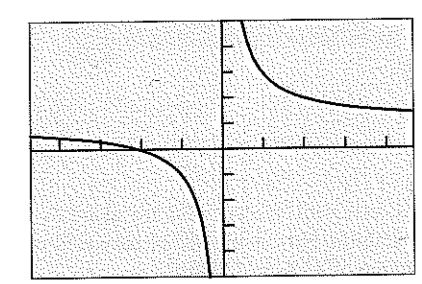
In Exercises 19–22, two possible solutions to the equation $f(x) = 0$ are listed. Use the given graph of $y = f(x)$ to decide which, if any, are extraneous.

19. $x = -5$ or $x = -2$



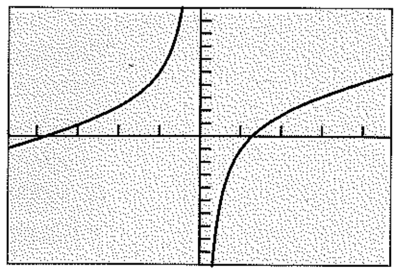
[–10, 8.8] by [–5, 5]

20. $x = -2$ or $x = 3$



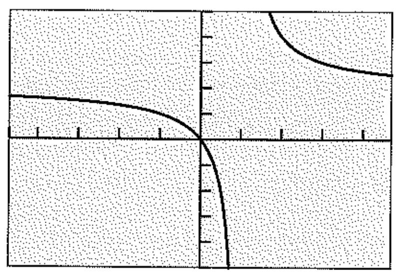
[–4.7, 4.7] by [–5, 5]

21. $x = -2$ or $x = 2$



$[-4.7, 4.7]$ by $[-10, 10]$

22. $x = 0$ or $x = 3$



$[-4.7, 4.7]$ by $[-5, 5]$

In Exercises 23–30, solve the equation.

23. $\dfrac{2}{x-1} + x = 5$

24. $\dfrac{x^2 - 6x + 5}{x^2 - 2} = 3$

25. $\dfrac{x^2 - 2x + 1}{x + 5} = 0$

26. $\dfrac{3x}{x+2} + \dfrac{2}{x-1} = \dfrac{5}{x^2 + x - 2}$

27. $\dfrac{4x}{x+4} + \dfrac{5}{x-1} = \dfrac{15}{x^2 + 3x - 4}$

28. $\dfrac{3x}{x+1} + \dfrac{5}{x-2} = \dfrac{15}{x^2 - x - 2}$

29. $x^2 + \dfrac{5}{x} = 8$

30. $x^2 - \dfrac{3}{x} = 7$

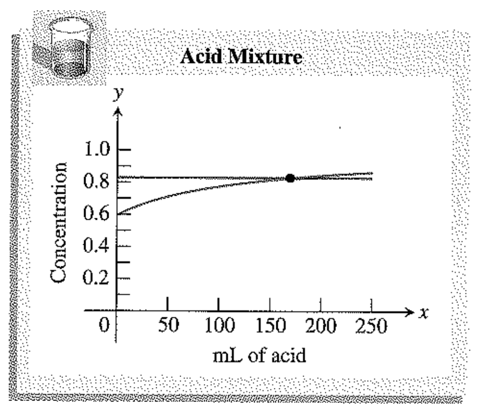
31. Acid Mixture Suppose that x mL of pure acid are added to 125 mL of a 60% acid solution. How many mL of pure acid must be added to obtain a solution of 83% acid?

(a) Explain why the concentration $C(x)$ of the new mixture is

$$C(x) = \frac{x + 0.6(125)}{x + 125}.$$

(b) Suppose the viewing window in the figure is used to find a solution to the problem. What is the equation of the horizontal line?

(c) Writing to Learn Write and solve an equation that answers the question of this problem. Explain your answer.



32. Acid Mixture Suppose that x mL of pure acid are added to 100 mL of a 35% acid solution.

(a) Express the concentration $C(x)$ of the new mixture as a function of x.

(b) Use a graph to determine how much pure acid should be added to the 35% solution to produce a new solution that is 75% acid.

(c) Solve (b) algebraically.

33. Breaking Even Mid Town Sports Apparel, Inc., has found that it needs to sell golf hats for \$2.75 each in order to be competitive. It costs \$2.12 to produce each hat, and it has weekly overhead costs of \$3000.

(a) Let x be the number of hats produced each week. Express the average cost (including overhead costs) of producing one hat as a function of x.

(b) Solve algebraically to find the number of golf hats that must be sold each week to make a profit. Support your answer graphically.

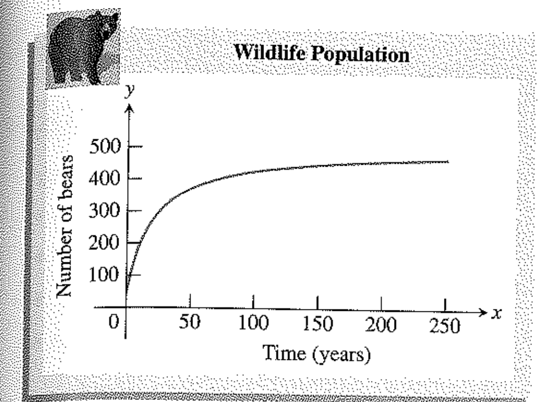
(c) Writing to Learn How many golf hats must be sold to make a profit of \$1000 in 1 week? Explain your answer.

34. Bear Population The number of bears at any time t (in years) in a federal game reserve is given by

$$P(t) = \frac{500 + 250t}{10 + 0.5t}.$$

(a) Find the population of bears when the value of t is 10, 40, and 100.

(b) Does the graph of the bear population have a horizontal asymptote? If so, what is it? If not, why not?



(c) Writing to Learn According to this model, what is the largest the bear population can become? Explain your answer.

35. Minimizing Perimeter Consider all rectangles with an area of 182 ft^2. Let x be the length of one side of such a rectangle.

(a) Express the perimeter P as a function of x.

(b) Find the dimensions of the rectangle that has the least perimeter. What is the least perimeter?

36. Group Activity Page Design Hendrix Publishing Co. wants to design a page that has a 0.75-in. left border, a 1.5-in. top border, and borders on the right and bottom of 1-in. They are to surround 40 in.^{2} of print material. Let x be the width of the print material.

(a) Express the area of the page as a function of x.

(b) Find the dimensions of the page that has the least area. What is the least area?

37. Industrial Design Drake Cannery will pack peaches in 0.5-L cylindrical cans. Let x be the radius of the can in cm.

(a) Express the surface area S of the can as a function of x.

(b) Find the radius and height of the can if the surface area is 900 cm^2.

38. Group Activity Designing a Swimming Pool Thompson Recreation, Inc., wants to build a rectangular swimming pool with the top of the pool having surface area 1000 ft^2. The pool is required to have a walk of uniform width 2 ft surrounding it. Let x be the length of one side of the pool.

(a) Express the area of the plot of land needed for the pool and surrounding sidewalk as a function of x.

(b) Find the dimensions of the plot of land that has the least area. What is the least area?

39. Resistors The total electrical resistance R of two resistors connected in parallel with resistances R_1 and R_2 is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

One resistor has a resistance of 2.3 ohms. Let x be the resistance of the second resistor.

(a) Express the total resistance R as a function of x.

(b) Find the resistance of the second resistor if the total resistance of the pair is 1.7 ohms.

40. Designing Rectangles Consider all rectangles with an area of 200 m^2. Let x be the length of one side of such a rectangle.

(a) Express the perimeter P as a function of x.

(b) Find the dimensions of a rectangle whose perimeter is 70 m.

41. Swimming Pool Drainage Drains A and B are used to empty a swimming pool. Drain A alone can empty the pool in 4.75 h. Let t be the time it takes for drain B alone to empty the pool.

(a) Express as a function of t the part D of the drainage that can be done in 1 h with both drains open at the same time.

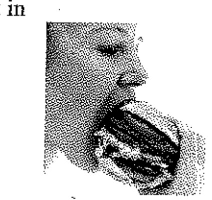
(b) Find graphically the time it takes for drain B alone to empty the pool if both drains, when open at the same time, can empty the pool in 2.6 h. Confirm algebraically.

42. Time-Rate Problem Josh rode his bike 17 mi from his home to Columbus, and then traveled 53 mi by car from Columbus to Dayton. Assume that the average rate of the car was 43 mph faster than the average rate of the bike.

(a) Express the total time required to complete the 70-mi trip (bike and car) as a function of the rate x of the bike.

(b) Find graphically the rate of the bike if the total time of the trip was 1 h 40 min. Confirm algebraically.

43. Fast Food Sales The total amount in sales in billions of dollars by fast food business for several years is given in Table 2.20. Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth. A model for the data is given by



$$y = 120 - \frac{500}{x + 8}$$

(a) Graph the model together with a scatter plot of the data.

(b) Use the model to estimate the amount of sales by fast food business in 2005.

Table 2.20 Fast Food Sales

Year	Amount (in billions)
1992	70.6
1993	74.9
1994	78.5
1995	82.5
1996	85.9
1997	88.8
1998	92.5
1999	97.5
2000	101.4
2001	105.5

Source: Technomic, as reported in USA Today July 3–4, 2002.